1. Two particles A and B have position vectors $\mathbf{r}_A$ metres and $\mathbf{r}_B$ metres at time t seconds, where

$$\mathbf{r}_A = t^2\mathbf{i} + (3t - 1)\mathbf{j} \text{ and } \mathbf{r}_B = (1 - 2t^2)\mathbf{i} + (3t - 2t^2)\mathbf{j}, \text{ for } t \geq 0.$$

(a) Find the values of t when A and B are moving with the same speed. [5]

(b) Show that the distance, d metres, between A and B at time t satisfies

$$d^2 = 13t^4 - 10t^2 + 2.$$ [3]

(c) Hence find the shortest distance between A and B in the subsequent motion. [6]

END OF QUESTION paper

Mark scheme

Question			Answer/Indicative content	Marks	Guidance	
1		a	$\dot{\mathbf{r}}_A = 2t\mathbf{i} + 3\mathbf{j}$	B1 (AOs 1.1)		
			$\dot{\mathbf{r}}_B = -4t\mathbf{i} + (3-4t)\mathbf{j}$	B1 (AOs 1.1)		
			$(2t)^2 + 9 = (-4t)^2 + (3-4t)^2$	M1 (AOs 3.1a)	$\lvert\dot{\mathbf{r}}_A\rvert = \lvert\dot{\mathbf{r}}_B\rvert$ with/without square root	
			$7t^2 - 6t = 0 \Rightarrow t = \ldots$	M1 (AOs 1.1)	Expand and attempt to solve quadratic in t (to obtain two solutions)	
			$t = 0$ or $t = \frac{6}{7}$	A1 (AOs 1.1) [5]	Both values of t must be given	
		b	$\mathbf{r}_A - \mathbf{r}_B = (3t^2 - 1)\mathbf{i} + (-1 + 2t^2)\mathbf{j}$	*M1 (AOs 3.1a)	Consider $\pm(\mathbf{r}_A - \mathbf{r}_B)$	Condone one sign error
			$d^2 = (3t^2 - 1)^2 + (-1 + 2t^2)^2$	dep*M1 (AOs 1.1)	Use of $d^2 = \lvert\mathbf{r}_A - \mathbf{r}_B\rvert^2$	
			$= 9t^4 - 6t^2 + 1 + 4t^4 + 1 = 13t^4 - 10t^2 + 2$	A1 (AOs 2.2a) [3]	**AG** Expand correctly to given answer	Must show at least one intermediate step
		c	$\frac{\mathrm{d}}{\mathrm{d}t}(d^2) = 52t^3 - 20t$	B1 (AOs 3.1a)		Ignore any mention of negative values of t
			$\frac{\mathrm{d}}{\mathrm{d}t}(d^2) = 0 \Rightarrow t = \ldots$	*M1 (AOs 2.1a)	Set their derivative = 0 and solve for t	
			$t = 0$ and $t = \sqrt{\frac{5}{13}}$	A1 (AOs 1.1)	Both values correct; accept 0.620…	
			Test nature of stationary point with correct value(s) of t	B1 (AOs 2.1)	e.g. $\frac{\mathrm{d}^2}{\mathrm{d}t^2}(d^2) = 156t^2 - 20 > 0$	
				dep*M1 (AOs		

			Substsitute their non-zero t into d or d^2 $d = \frac{1}{\sqrt{13}}$ or 0.277	1.1) A1 (AOs 2.2a) [6]	when $t^2 = \frac{5}{13}$ so minimum Dependent on all previous marks	Or any other valid method 0.277350…
			Total	14		